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EXCHANGE RATE BANDS AND OPTIMAL MONETARY ACCOMMODATION UNDER A DIRTY FLOWT

FLOAT

by Roel M.W.J. Beetsma
and Frederick van der Ploeg

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EXCHANGE RATE BANDS AND OPTIMAL MONETARY ACCOMMODATION UNDER A DIRTY FLOAT*

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ABSTRACT

This paper studies regimes of managed exchange rates for a small open economy with an integrated capital market, rational expectations in financial markets, sluggish nominal wages and prices, and supply shocks that follow a Brownian motion. Each regime can be characterised by the degree to which price shocks are accommodated and the width of the exchange rate band. Special cases of monetary accommodation are a peg, a clean float and a PPP exchange rate rule. First, the optimal degree of monetary accommodation of price shocks is analysed when there is no exchange rate band. Given that the welfare loss is a weighted sum of the asymptotic variances of output and of consumer prices, monetary accommodation is particularly strong when the authorities care relatively more about full employment than price stability. More flexible labour markets induce right-wing governments to move towards a cleaner float and left-wing governments towards a PPP exchange rate rule. Second, the effects of exchange rate bands and the accompanying inframarginal interventions is examined when allowance is made for intramarginal interventions as well. Such a set-up can, in contrast to the pioneering Krugman model, explain the observed hump-shaped unconditional density functions of EMS exchange rates.

Keywords: Optimal monetary accommodation, dirty floating, exchange rate bands, PPP exchange rate rules, exchange rate peg, interest rates, Brownian motion, price shocks, Ornstein-Uhlenbeck process, stochastic simulation.

JEL code: E0, F3, F4

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1 Introduction

The seminal paper of Krugman (1991) was the first to explicitly analyse the effects of exchange rate bands on exchange rate behaviour. When fundamentals follow a Brownian motion with drift, the solution for the exchange rate is a S-shaped function of fundamentals. Credible commitment of the central bank to defend the exchange rate exerts a stabilising influence on the exchange rate, even before intervention at the boundaries of the band takes place. The reason is that a weak currency goes with expectations of future monetary contractions in order to defend the band, and this strengthens the currency today.

Froot and Obstfeld (1991a) note that Krugman's model corresponds to a stochastic monetary model of a small open economy with perfectly flexible nominal wages and prices, full employment, uncovered interest parity and purchasing power parity. Klein (1990) demonstrates that narrower bandwidths in a classical model must lead to greater stabilisation of nominal and real exchange rates and of output. Svensson (1991a) shows that for narrow exchange rate bands, the asymptotic unconditional variability of the interest rate differential increases in the bandwidth, but for wide bands it slowly decreases in the bandwidth. The instantaneous variability of the interest rate differential monotonically decreases in the bandwidth. Note that the instantaneous standard deviation of the interest rate differential under a narrow band is high and increases when the band narrows, which sharply contrasts with a regime of pegged exchange rate and a zero interest rate differential. This observation may have some implications for the transition from the EMS to the EMU. Svensson (1991a) also shows that the asymptotic unconditional density function of nominal exchange rates is U-shaped, i.e. more mass is located towards the edges of the band than with the uniform distribution.[1]

The applicability of the Krugman (1991) model is severely limited due to the assumptions of price flexibility and purchasing power parity. Miller and Weller (1989, 1991) therefore extend the analysis to allow for sluggish nominal wage dynamics, transitory unemployment and imperfect substitution between home and foreign goods. Within this richer environment, one is able to distinguish between real and nominal exchange rates. This seems an essential extension, because it is difficult to make a welfare case for the

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[1] Svensson (1991b) derives properties of the term structure of interest rates, which may be used to test the validity of the Krugman (1991) target zone model of exchange rates.

presence of exchange rate bands in a classical model with full employment. If wages and prices are perfectly flexible, one may as well have a peg or one currency and reap all the benefits of an optimum currency area.

However, in the presence of nominal wage sluggishness and transitory unemployment, one wonders whether the central bank has an interest in monetary accommodation of wage and price shocks. Dornbusch (1982) and Alogoskoufis (1991) examined the effects of monetary accommodation, and of PPP exchange rate rules in particular[2], on macroeconomic stability in a framework of dirty floating. However, no work has been done on the effects of monetary accommodation within nominal exchange rate bands.[3]

The main objective of this paper is thus to relax the assumptions of full employment and purchasing power parity prevalent in most of the literature on exchange rate bands and consequently to analyse the effects of monetary accommodation and of exchange rate bands on the exchange rate, the interest rate, the consumption price index, unemployment and output. A coefficient of monetary accommodation equal to zero and to one correspond to a clean float and a PPP exchange rate rule, respectively. One particular value of the accommodation coefficient corresponds to a peg. Given a welfare loss function which is a weighted sum of the asymptotic variances of the consumption price index and output, the paper also analyses what the optimal degree of monetary accommodation and bandwidth are. Typically, a peg, a clean float or a PPP exchange rate rule are sub-optimal relative to a more general dirty float. The imposition of a band on the nominal exchange rate corresponds to a nonlinear accommodation rule, because the accommodation coefficient makes a discrete jump when prices (or the exchange rate) move outside a band. Within the context of a linear model

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[2] Increased nominal exchange rate indexation ensures more stability in the real exchange rate and the levels of demand and employment, but on the other hand it amplifies the effects of wage disturbances on prices. Alogoskoufis (1991) extends Dornbusch (1982) by highlighting the effects of monetary accommodation and exchange rate accommodation on the forward-looking behaviour of wage-setters and price-setters, and suggests that the higher degree of persistence in inflation arises from a higher degree of accommodation in floating exchange rate regimes. For the model discussed in this paper, the PPP exchange rate rule corresponds to 100% monetary accommodation of price and wage shocks.

[3] Lewis (1990) analyses a specific type of intramarginal interventions (i.e. at each point of time there is a non-zero probability that the Brownian motion of the fundamental is stopped) within the context of a monetary model with full employment.

and quadratic preferences, no welfare case can be made for such an exchange rate band as long as intramarginal accommodation is feasible.

Some other interesting results emerge from this analysis and provide an answer to an important empirical puzzle for the EMS countries. Figure 1 shows that the histograms approximating the unconditional density functions of the guilder-Deutschemark and the franc-Deutschemark rate during the EMS period are clearly hump-shaped, whereas Svensson (1991a) predicts on the basis of the Krugman (1991) model that these density functions should be U-shaped. For more recent years, the hump-shaped character of the histograms of EMS exchange rates is even more pronounced. Theory predicts that most of the probability mass is located near the edges of the band, whereas EMS data strongly indicate that the mass is concentrated near the middle of the band. The extension of the Miller-Weller-Krugman model presented in this paper offers an explanation of this empirical puzzle, that is the hump-shaped density function is a consequence of a relatively high degree of mean reversion induced by nominal wage sluggishness and monetary accommodation. This explanation of hump-shaped density functions seems much more satisfactory than one based on the anticipation of repeated realignments, because realignments do not seem very likely for the EMS anymore.[4]

Section 2 studies regimes of pegged exchange rates and of dirty floating for a small open economy with sluggish nominal wages, transitory unemployment, uncovered interest parity, imperfect substitution between home and foreign goods, and rational expectations. For low degrees of monetary accommodation, a permanent change in the money supply induces the exchange rate to overshoot, whereas for high degrees of accommodation, the exchange rate undershoots. The dividing line corresponds to that specific degree of accommodation which corresponds to pegged exchange and interest rates.

Section 3 shows that, when shocks to the Phillips-curve follow a Wiener process, the asymptotic variances of output and the real exchange rate decline monotonically, whereas the asymptotic variances of production and consumption prices increase mononotically with the degree of monetary accommodation. The volatility of the nominal exchange rate first declines

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[4] An alternative answer to the puzzle is provided by Bertola and Caballero (1992) and Bertola and Svensson (1991) who take account of the anticipation of repeated realignments of the central parity. However, for recent years this does not seem a very plausible answer as realignments occur rarely nowadays.

as accommodation increases and completely disappears as one approaches a peg, but then increases and approaches infinity as the money supply approaches a PPP exchange rate rule. The asymptotic variance of the interest rate declines and approaches zero as monetary accommodation increases from zero to the value that ensures a peg. Subsequently, it first rises and then falls to zero as the degree of accommodation approaches the value corresponding to a PPP exchange rate rule. The optimal degree of accommodation to price shocks is high when the authorities care relatively more about full employment than price stability. More flexible labour markets induce right-wing governments to have a lower optimal accommodation coefficient, i.e. move towards a cleaner float, and induce left-wing governments to have a higher optimal accommodation coefficient, i.e. move towards a PPP exchange rate rule.

Section 4 gives an explicit solution for the exchange rate when there is intramarginal monetary accommodation within an exchange rate band for the special case that wages adjust instantaneously to clear the labour market. Shocks to the full-employment level of output follow a Brownian motion. The familiar S-shaped solution for the exchange rate as a function of fundamentals emerges. When the accommodation coefficient approaches the value corresponding to a peg, the S-shape becomes less pronounced and the implicit band on prices becomes wider. The unconditional density function for the exchange rate in this classical model must be U-shaped.

To explain the hump-shaped density functions found for EMS exchange rates, one must allow for a stronger form of mean reversion in the fundamentals. Section 5 shows that this can be achieved when one extends the model to allow for nominal wage sluggishness and transitory Keynesian unemployment. It then follows that the solution for the exchange rate is an inverted S-shaped function of the price level for low degrees of accommodation, and a regular S-shaped function for high degrees of accommodation. Section 5 also discusses how the asymptotic variances of prices, output, the interest rate and the exchange rate depend on the degree of accommodation, and contrasts them with the values under a peg, a pure float and a PPP exchange rate rule. This exercise suggests that the reason why some EMS countries have a hump-shaped rather than a U-shaped unconditional density function of the exchange rate is an accommodation coefficient close to the one corresponding to a peg. Hump-shaped density functions may thus be due to EMS countries engaging in intramarginal interventions (i.e. when the exchange rate is inside its band) as well as inframarginal interventions (when the exchange rate is on one of the boundaries of its band) to stabilise the exchange rate.

It is difficult to make a welfare case for exchange rate bands on stabilisation grounds alone. Section 6 therefore briefly discusses the welfare loss from having exchange rate bands within the context of a linear model and quadratic preferences, that is the welfare loss from having a nonlinear rather than a linear accommodation rule. It also discusses some second-best issues, such as what is the optimal bandwidth for a given degree of monetary accommodation and how does the optimal degree of accommodation change when the bandwidth narrows. Section 7 concludes the paper.

2 Unemployment and expectations in a small open economy

2.1 The model

Most of the existing models of exchange rate bands are based on the classic work of Krugman (1991), and adopt the assumptions of flexible wages and prices, full employment and purchasing power parity. However, for policy purposes it is essential to allow for sticky nominal wages and transitory unemployment. Otherwise, it is very difficult to make a case for monetary accommodation or to provide a rationale for exchange rate bands. It is also important to allow for imperfect substitution between home and foreign goods, so that one can allow for the effects of the real exchange rate on aggregate demand. Miller and Weller (1989, 1991) therefore extend the analysis of Krugman (1991) to the familiar exchange rate overshooting model of Dornbusch (1976). Here a stochastic version of such a model of a small open economy is extended to allow for regimes of managed exchange rates and for monetary accommodation of price shocks. The resulting model may be summarised by the following equations:

$$y = - \eta \ (i - \pi) + \delta \ (e + p^{*} - p), \ \eta>0, \ 0<\delta<1 \tag{2.1}$$

$$m - p = y - \lambda \ i, \ \lambda>0 \tag{2.2}$$

$$dp = \phi \ (y - y^{F}) \ dt + \pi \ dt + \sigma dz, \ dz \sim IN(0,dt), \ \phi>0 \tag{2.3}$$

$$E(de) = (i - i^{*}) \ dt \tag{2.4}$$

$$q = (1-\alpha)\ p + \alpha\ (p^* + e), \quad 0<\alpha\leq\tfrac{1}{2} \tag{2.5}$$

$$m = \mu + \beta\ (p - \mu) = (1-\beta)\ \mu + \beta\ p, \quad \beta<1 \tag{2.6}$$

where m, y, y^F, p, p^*, q, e and z denote logarithms of the nominal money supply, the level of aggregate demand, the full-employment level of output, the home price level, the foreign price level, the consumer price index (CPI), the nominal exchange rate (price of one unit of foreign currency in terms of domestic currency units) and a supply shock, respectively, i and i^* denote the home and foreign nominal interest rate, respectively, and π denotes the rate of core inflation. All variables are expressed as deviations from original steady-state values. The foreign variables are non-stochastic and can be normalised to zero.

Equation (2.1) is the IS-curve and shows that aggregate demand increases when the real interest rate declines or the real exchange rate depreciates. The real interest rate is simply the nominal interest rate minus the core inflation rate. Equation (2.2) is the LM-curve and says that the velocity of circulation increases with the nominal interest rate. Equation (2.3) is the Phillips-curve which shows that inflation in wages and producer prices occurs when there is a shortage of labour, and that deflation occurs when there is unemployment. The speed at which the labour market clears, i.e the degree of labour market flexibility, is determined by the parameter ϕ. The full-employment level of output can be normalised to zero ($y^F=0$). Supply shocks (z) follow a Brownian motion and correspond to positive shocks to nominal wages; z follows an independent Wiener process with zero mean and instantaneous variance equal to σ. Producer prices are a constant mark-up on unit labour costs, so that supply shocks may be interpreted as negative shocks to labour productivity. The money supply is stable in the steady state, so core inflation (π) being the expected steady-state rate of inflation is assumed to be zero.[5]

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[5] The advantage of this simple specification is that one can unambigiously determine that the system is saddlepath stable. For example, if aggregate demand depends on the real consumption interest rate, i.e. the nominal interest rate minus the rationally expected change in the CPI, there is a possibility of an unstable spiral. In other words, higher inflation depresses the real interest rate, boosts aggregate demand and thus induces even higher inflation. The use of core inflation in the definition of the real interest rate avoids these indeterminacies and simplifies the algebra.

Equation (2.4) is the uncovered interest parity condition. Risk-neutral arbitrage ensures that an interest differential in favour of the domestic country can only be sustained if the currency is expected to depreciate in the future, i.e. if the currency is currently over-valued. Equation (2.5) shows the CPI as a weighted average of domestic prices and foreign prices (expressed in home currency units). The share of imports in total expenditures corresponds to α. Finally, equation (2.6) is the feedback rule for the monetary authorities where μ is the exogeneous or long-run component of the nominal money supply and β is the accommodation coefficient. The reaction rule says that, when prices exceed their long-run value (i.e. the exogeneous component of the money supply), the monetary authorities accommodate and raise the money supply. If β is close to unity, the central bank accommodates almost all excessive demands for higher wages and prices, whilst if β is close to zero, the central bank does not give in to such demands.

2.2 Pegged exchange rates

Under a regime of pegged exchange rates, say a **peg**, the monetary authorities use unsterilised interventions in the foreign exchange market to stabilise and fix the nominal exchange rate. For example, when the foreign interest rate exceeds the domestic interest rate, there is an incipient capital outflow and pressure for the currency to depreciate. The central bank defends the exchange rate by selling foreign reserves and buying its own currency, and as a result the money supply falls until the domestic interest rate is pushed up to the level of the foreign interest rate. Similarly, when home prices increase or foreign prices decrease, there is pressure on the currency to depreciate and the central bank defends the exchange rate by selling foreign reserves and contracting the money supply.

A peg corresponding to $e=e_p$ implies that the domestic interest rate is anchored to the foreign interest rate, $i=i^*$, and that an independent domestic monetary policy is infeasible:

$$m = (1-\delta)\ p + \delta\ (p^* + e_p) - (\eta+\lambda)\ i^* \equiv (\quad \beta)\ (p - \mu) + \mu. \tag{2.7}$$

Hence, a regime of pegged exchange rates corresponds to a very specific form of monetary accommodation, namely $\beta=1-\delta$ and $\mu=p^*+e_p-(\eta+\lambda)i^*/\delta$. Upon substitution of these results into equations (2.1) and (2.3), one finds that prices follow an Orstein-Uhlenbeck process (Karlin and Taylor, 1981, p. 172):

$$dp = - \phi \; [\eta \; i^* + \delta \; (p - p^* - e_p)] \; dt + \sigma \; dz. \tag{2.8}$$

The asymptotic distribution of this process has:

$$\text{mean}(p) = p^* + e_p - (\eta/\delta) i^* = \mu \quad \text{and} \quad \text{var}(p) = \sigma^2/(2\phi\delta).$$

One can show that $\text{mean}(y)=0$, $\text{mean}(q)=p^*+e_p-(1-\alpha)(\eta/\delta)i^*$, $\text{var}(y)=\sigma^2\delta/(2\phi)$ and $\text{var}(q)=(1-\alpha)^2\sigma^2/(2\phi\delta)$. An increase in labour market flexibility (ϕ) reduces both the variance of prices and the variance of output. In a classical model with full employment the variance of prices and output tend to zero. An increase in the sensitivity of aggregate demand to relative prices (δ) reduces the variance of prices, but increases the variance of output. As far as forecasting is concerned, one has:

$$E[p(t+s) \;/\; p(t)=x] = \text{mean}(p) \; [1-\exp(-\phi\delta s)] + x \; \exp(-\phi\delta s), \quad s>0$$

$$\text{var}[p(t+s) \;/\; p(t)=x] = [1 - \exp(-2\phi\delta s)] \; \text{var}(p), \quad s>0.$$

Clearly, prices and the other variables follow mean-reverting processes. With the aid of Bergstrom (1984, pp. 1154-1155), one can solve for the inflation rate over, say, a unit time interval:

$$\text{var}(\Delta p) \equiv \text{var}[p(t)-p(t-1)] = 2 \; [1 - \exp(-\phi\delta)] \; \text{var}(p)$$

and similarly for $\text{var}(\Delta q)$. Note that when the labour market is completely inflexible, the variance of the price level is infinite whilst the variance of the rate of inflation equals (the product of) the instantaneous variance of supply shocks (and the time interval). When the labour market is perfectly flexible, the variance of both the price level and the rate of inflation are zero.

2.3 Monetary accommodation under a dirty float

In general, the monetary authorities follow a linear feedback rule such as (2.6). An advantage of such a simple rule is that it is easily understood by the market. A regime of floating exchange rates with a rule for monetary accommodation such as (2.6) will be referred to as a **dirty float**. The reduced form equations of the model under a dirty float are:

$$dp = \phi(\eta+\lambda)^{-1}\{-[\delta\lambda+\eta(1-\beta)]\ p + \delta\lambda\ (e+p^*) + \eta(1-\beta)\ \mu\}\ dt + \sigma dz \tag{2.9}$$

$$Ede = \{(\eta+\lambda)^{-1}[(1-\beta-\delta)\ p + \delta\ (e+p^*) - (1-\beta)\ \mu] - i^*\}\ dt. \tag{2.10}$$

The price level is a predetermined, backward-looking variable, whilst the exchange rate is a non-predetermined, forward-looking variable which jumps if private agents suddenly anticipate a change in future policy. The rational expectations equilibrium must therefore be a stable saddlepath solution. This requires one eigenvalue with a negative real part and one eigenvalue with a positive real part, which is ensured as long as $\beta<1$ holds. To find the unique, non-explosive rational expectations solution under an unrestricted dirty float, one postulates a linear saddlepath:

$$e - \text{mean}(e) = \omega\ [p - \text{mean}(p)] \tag{2.11}$$

where $\text{mean}(p)=\mu+\lambda(1-\beta)^{-1}i^*$ and $\text{mean}(e)=\text{mean}(p)-p^*+\eta\delta^{-1}i^*$. Upon substitution of (2.11) into (2.9) and (2.10) and equating coefficients, one finds:

$$\omega = \left(\frac{\phi\eta(1-\beta)+\delta(1+\lambda\phi)-\sqrt{[\phi\eta(1-\beta)+\delta(1+\lambda\phi)]^2+4\lambda\phi\delta(1-\beta-\delta)}}{2\lambda\phi\delta}\right).$$

In fact, there are two solutions for ω. However, the other solution can be ruled out because it does not satisfy the requirement that the adjustment of prices along the saddlepath must be a stable process. Stability requires that, if $\beta<1-\delta$ and thus $\omega<0$, $\omega<1-(1-\beta)\delta^{-1}$ and, if $\beta>1-\delta$ and thus $\omega>0$, $\omega>1-(1-\beta)\delta^{-1}$. Clearly, the above solution for ω is stable.

For low degrees of monetary accommodation, i.e. $\beta<1-\delta$, there is a negative correlation between nominal exchange rates and prices, $\omega<0$, and for high degrees of monetary accommodation, $\beta>1-\delta$, there is a positive correlation between nominal exchange rates and prices along the saddlepath, $\omega>0$. The turning point is the degree of monetary accommodation that corresponds to a peg, $\beta=1-\delta$. The saddlepath solutions for these two cases of low and high degrees of monetary accommodation are presented in Figure 2. Given that $\delta \geq \frac{1}{2}$, it can be shown that the smallest value of ω exceeds -1.

The steady state corresponds to the mean of the asymptotic distributions, i.e. omitting the effects of i^* one has $\text{mean}(p) = \text{mean}(e+p^*) = \mu$. In the long run relative purchasing power parity holds and money is neutral. The effect of a 1% increase in the exogeneous (or long-run) component of the money supply, μ, raises prices and exchange rates by exactly 1%, independent of the degree of monetary accommodation in the short run.

The full-employment locus requires, in the absence of shocks, stable prices ($dp=0$) and is steeper than 45^0, because its slope $(1+\eta(1-\beta)(\lambda\delta)^{-1})$ is greater than unity. A 1% increase in the price level reduces aggregate demand through an appreciation of the real exchange rate and through a contraction of real money balances, hence to ensure full employment the exchange rate must depreciate by more than 1%. The effect on real money balances is less when the accommodation coefficient is high, so that as β increases the full-employment locus becomes flatter and tilts towards the 45^0 line.

The interest-parity locus requires that the market expects a stable exchange rate ($Ede=0$). An increase in the price level induces on the one hand a monetary contraction and an increase in the interest rate, less so if there is a high degree of monetary accommodation, and on the other hand an appreciation of the real exchange rate, a fall in aggregate demand and a fall in the interest rate. The latter (former) effect is more likely to dominate when β is high (low), in which case a depreciation (appreciation) of the exchange rate is required to push up (depress) the interest rate back to the foreign rate. More precisely, the slope of the interest-parity locus $(-(1-\beta-\delta)/\delta)$ is negative for low degrees of monetary accommodation ($\beta<1-\delta$), whilst it is positive and less than unity for high degrees of monetary accommodation ($\beta>1-\delta$).

The qualitative properties of the rational expectations equilibrium solution are very much dependent on the degree of monetary accommodation. For example, Figure 2 shows that an increase in the exogeneous or long-run component of the money supply induces overshooting of the exchange rate (cf., Dornbusch, 1976) for low degrees of accommodation. The market expects an interest rate differential in favour of abroad and thus over time a gradually appreciating exchange rate, so that the exchange rate must on impact over-react. However, for high degrees of accommodation, the exchange rate undershoots on impact. A gradual increase in prices is accompanied by a gradual expansion of the money supply, so that the anticipated gradual rise in interest rates is less pronounced. Consequently, the exchange rate depreciates less and the interest rate falls less on impact thus leading to undershooting rather than overshooting of the exchange rate. Also, the steady-state value of the exchange rate increases by more when there is more accommodation, thus making undershooting more likely.

The special case of a peg corresponds to $\beta=1-\delta$ (see section 2.2), which implies $\omega=0$. A peg ensures stable exchange and interest rates, hence the interest-parity locus and the saddlepath (SS) are horizontal and coincide. In general, the saddlepath is for $\beta<1-\delta$ downward-sloping and becomes flatter as the accommodation coefficient is increased.

The special case of a regime of unrestricted floating exchange rates corresponds to an exogeneous supply of money and no accommodation ($\beta=0$). This will be referred to as a **clean float**. This case always leads to overshooting of the exchange rate after a monetary expansion ($\omega<0$). In general, one has a dirty float which ensures that the exchange rate adjusts instantaneously to ensure equilibrium on the balance of payments.

Figure 3 summarises the dependence of the slope of the saddlepath, ω, on the accommodation coefficient. Basically, ω is an increasing function of β[6], and is negative for low and positive for high values of β. Figure 3 also shows the dependence of the slope of the saddlepath on the degree of labour market flexibility, ϕ. As ϕ increases, the relationship between ω and β pivots around the peg ($\beta=1-\delta$) towards the horizontal axis. In the extreme case of a classical model with full employment, $\phi\to\infty$, one has a horizontal saddlepath, $\omega=0$, and no transitional dynamics.

2.4 PPP exchange rate rule

The special case of full accommodation of price shocks ($\beta=1$) means that the full-employment, the interest-parity locus and the saddlepath coincide with the 45^0 line ($\omega=1$). When $\beta=1$, one has $m=p$ and the expected change in the real exchange rate is from (2.9)-(2.10) given by:

$$\mathrm{E}\ d(p^*+e-p) = (1-\phi\lambda)\delta(\eta+\lambda)^{-1}\ (p^*+e-p)\ dt - i^*\ dt. \tag{2.12}$$

[6] In fact, this is the case if $\delta\geq0.5$, $\phi\eta\leq1$ and $\lambda\phi\leq1$.

If the inequality $\phi\lambda<1$ is satisfied, the rational expectations equilibrium solution for the real exchange rate is that it must jump to:

$$p^* + e - p = (\eta+\lambda)\delta^{-1}(1-\phi\lambda)^{-1}\ i^*. \tag{2.12'}$$

Full accommodation of price shocks thus eliminates under rational expectations all transitional dynamics. It is thus clear that 100% accommodation of price and wage shocks corresponds to a rule which ensures that the domestic interest rate is pegged to the foreign interest rate, that the real exchange rate is fixed, and thus that employment and output are at their natural rates. This special case corresponds to a **PPP exchange rate rule** of the type discussed by Dornbusch (1982) and Alogoskoufis (1991), and induces non-stationary nominal prices and exchange rates ($\text{var}(p)=\text{var}(e)=\infty$).

3 Optimal monetary accommodation under a dirty float

This paper assesses the case for managed exchange rates. In order to do this, assume that the monetary authorities minimise the following welfare loss function:

$$W = \text{var}(y) + \gamma\ \text{var}(q), \quad \gamma>0 \tag{3.1}$$

where W denotes the welfare loss. More conservative and more independent central bankers are likely to care more about CPI stability rather than full employment and thus have a higher value of γ. Of course, the optimal outcomes for β are unaffected when the authorities minimise expected squared deviations from some desired value.[7]

Since there are no bands on exchange rates, the underlying model is linear and the expected values of the variables correspond exactly to the solution of the deterministic model obtained by setting $\sigma=0$. Hence, the optimal value of β in the policy rule (2.6) is independent of the value of σ. This property is what is referred to as **certainty equivalence** and no longer holds once there are bands on exchange rates (see sections 4, 5 and 6).

Along the saddlepath solution, prices follow an Ornstein-Uhlenbeck

[7] The authorities could have minimised a discounted integral of squared deviations, but here an asymptotic welfare loss criterion is used in order not to get detracted by initial-value problems.

process:

$$dp = -\phi(\eta+\lambda)^{-1}[\delta\lambda(1-\omega)+\eta(1-\beta)]\ (p - \mu)\ dt + \sigma\ dz. \tag{3.2}$$

Hence, one has

$$\mathrm{var}(p) = (\eta+\lambda)\sigma^2/\{2\phi[\delta\lambda(1-\omega)+\eta(1-\beta)]\}$$

so that

$$\mathrm{var}(e) = \omega^2\ \mathrm{var}(p),$$
$$\mathrm{var}(e+p^*-p) = (1-\omega)^2\ \mathrm{var}(p),$$
$$\mathrm{cov}(e,p) = \omega\ \mathrm{var}(p),$$
$$\mathrm{var}(q) = [1-\alpha(1-\omega)]^2\ \mathrm{var}(p),$$
$$\mathrm{var}(y) = \{[\delta\lambda(1-\omega)+\eta(1-\beta)]/(\eta+\lambda)\}^2\ \mathrm{var}(p)$$

and

$$\mathrm{var}(i) = \{[1-\beta-\delta(1-\omega)]/(\eta+\lambda)\}^2\ \mathrm{var}(p).$$

The dependence of these asymptotic variances under a dirty float on β are sketched in Figures 4-9.

Since ω increases in β, it is clear that var(p) increases in β. In fact, with full accommodation of price shocks (β=1), i.e. a PPP exchange rate rule, var(p) tends to infinity. When labour markets are more flexible (ϕ higher) price volatility is less. The variance of nominal exchange rates first decreases with the accommodation coefficient, but once a peg has been reached increases with the accommodation coefficient. In fact, var(e) tends to infinity when all price shocks are fully accommodated and a PPP exchange rate rule is in place. Obviously, var(e)=0 for a peg (β=1-δ). However, the variance of real exchange rates typically declines with the accommodation coefficient and is only zero when there is full accommodation of price shocks and a PPP exchange rate rule (β=1). For low degrees of monetary accommodation, there is a negative correlation between nominal exchange rates and producer prices whilst for high degrees of monetary accommodation there is a positive correlation. It is also straightforward to establish that the variance of the CPI increases with the degree of accommodation. Furthermore, the variance of the CPI is always less than the variance of the producer price because ω<1.

The variance of output is zero when there is full accommodation of price shocks, i.e. a PPP exchange rate rule. Typically, the variance of output declines with the degree of monetary accommodation. The variance of nominal interest rates is zero both when there is a peg and when there is full accommodation of price shocks and a PPP exchange rate rule in place. For other degrees of accommodation, var(i) is positive.

The asymptotic variance of CPI inflation over a unit time interval can be written as:

$$\mathrm{var}(\Delta q) = 2\ \mathrm{var}(q)\ \{1 - \exp[-\phi[\delta\lambda(1-\omega)+\eta(1-\beta)]/(\eta+\lambda)]\} \tag{3.3}$$

Clearly, when labour markets clear instantaneously, var(Δq)=2var(q). In general, it can be shown that var(Δq) is an increasing function of the accommodation coefficient (β). Typically, the variance of inflation is less than the variance of the price level, especially for high degrees of accommodation.

From the point of view of optimal policy formulation, it is useful to have Figure 10 which shows a convex policy frontier characterising the trade-off between the variance of employment and the variance of the CPI (a second-order asymptotic Phillips-curve). Obviously, the optimal accommodation coefficient decreases when the weight given to CPI stability rather than full employment (γ) increases. Very right-wing governments who attach an extremely high value to CPI stability find a clean float (β=0) optimal. In general, it is optimal to have a dirty float.[8]

As the degree of labour market flexibility (ϕ) increases, both the variance of output and of the CPI diminish and thus the policy frontier moves towards the origin. For low degrees of accommodation (β<1-δ), the optimal accommodation coefficient decreases, and for high degrees of accommodation (β>1-δ), the optimal degree of accommodation increases.[9] The former case suggests that right-wing governments view a more flexible labour market as a reason for a cleaner float, whereas the latter case suggests that left-wing governments view this as a reason for a PPP exchange rate rule. Clearly, when wages and prices immediately adjust to ensure full employment ($\phi\rightarrow\infty$) var(y), var(p) and var(q) tend to zero so that the degree of monetary accommodation (as long as it is not complete, i.e. β<1) is irrelevant and the specific nature of the nominal exchange rate regime does not matter.

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[8] The welfare loss under a peg is independent of λ and η, since under a peg var(y) = δ^2var(p) = $\delta\sigma^2/(2\phi)$.

[9] The reason is that as ϕ increases, the ratio var(q)/var(y) increases when β<1-δ and decreases when β>1-δ.

An increase in the share of foreign goods in the consumption basket (α) decreases CPI variability (given that var(e)<var(p) must hold when $\beta<1$ and $\alpha<\frac{1}{2}$), but does not affect output variability. It is therefore clear that the optimal accommodation coefficient (β) increases in the degree of exposure to international trade (α).

4 Exchange rate bands, monetary accommodation and full employment

To facilitate a comparison with the results of Krugman (1991), Svensson (1991a) and Klein (1990), the model is first solved under the assumption that wages adjust instantaneously to clear the labour market ($\phi\rightarrow\infty$). Suppose that supply shocks affect the full-employment level of output rather than prices and that they have an instantaneous variance equal to σ', that is the full-employment level of output consists of a deterministic component (y^F) and a stochastic component (ϵ) which follows a Brownian motion without drift:

$$y = y^F + \epsilon, \quad d\epsilon = \sigma' \ dz, \quad dz \sim \text{IN}(0,\ dt). \tag{2.3'}$$

This classical specification of the labour market[10] replaces the Keynesian specification (2.3). The complete model includes (2.1), (2.2) and (2.4)-(2.6). The reduced form of this classical model is:

$$\begin{aligned} \text{E } de \ &= \eta^{-1} \ [\delta \ (e+p^*-p) \ - \ (y^F+\epsilon)] \ dt \ - \ i^* \ dt \\ &= [\eta(1-\beta)+\delta\lambda]^{-1} \ [\delta(1-\beta) \ (e+p^*-\mu) \ - \ (1-\beta-\delta) \ (y^F+\epsilon)] \ dt \ - \ i^* \ dt \\ &= [(e \ - \ f)/\lambda'] \ dt \qquad \text{or} \qquad e \ dt = f \ dt + \lambda' \ \text{E } de \end{aligned} \tag{4.1}$$

and

$$p = [\delta\lambda \ (p^* + e) + \eta(1-\beta) \ \mu \ - \ (\eta+\lambda) \ (y^F + \epsilon)]/[\eta(1-\beta)+\delta\lambda] \tag{4.2}$$

where

$$\lambda' \equiv [\eta(1-\beta)+\delta\lambda]/[\delta(1-\beta)] > 0$$

and

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[10] This specification of permanent shocks to the full-employment level of output corresponds closely to the format of Krugman (1991). One could alternatively maintain (2.3), but assume that $\sigma=\sigma'\phi$ and let both σ and ϕ tend to infinity keeping σ' constant. This alternative specification implies transitory shocks to the full-employment level of output.

$$f \equiv \mu - p^* + \lambda' i^* + [\delta(1-\beta)]^{-1} (1-\beta-\delta) (y^F + \epsilon) = f' + \epsilon'.$$

The fundamental (f) consists of two components. The first component (f') is a constant which increases in the long-run component of the money supply and the foreign interest rate, and decreases in the foreign price level. The second component ($\epsilon' \equiv (1-\beta-\delta)\epsilon/[\delta(1-\beta)]$) is due to Brownian motion in the supply of goods and has an instantaneous standard deviation given by $\sigma'' \equiv (1-\beta-\delta)\sigma'/[\delta(1-\beta)]$. When bubbles are excluded, it follows that the rational expectations solution of (4.1) is given by

$$e(t) = E_t \left[\int_{s=t}^{\infty} \exp(-(s-t)/\lambda') \, f(s) \, ds\right]/\lambda'. \tag{4.3}$$

Hence, the exchange rate is an average of present values of expected future fundamentals with ($1/\lambda'$) being the rate of discount. Under a dirty float without exchange rate bands, one has

$$e(t) = f(t) = f'(t) + \epsilon'(t) \tag{4.4}$$

as $E_t[f(s)]=f(t)$, $s>t$.

So far, the analysis has allowed for a linear rule for monetary accommodation. In practice, monetary authorities often restrict themselves to target zones for nominal exchange rates. For example, the EMS countries have committed themselves to keep exchange rates within a ±2.25% band around a central rate, except for the United Kingdom which has a ±6% band. Here allowance is made on the one hand for infinitesimal intervention at the upper boundary, e^U, and lower boundary of the band, e_L, to prevent the exchange rate going out of the band, and on the other hand for monetary accommodation according to the rule (2.6) within the band.

If one postulates a nonlinear solution of the form $e=\Omega(f)$ and applies Ito's Lemma (e.g. Harrison, 1985), one obtains:

$$e = \Omega(f) = f + \tfrac{1}{2} \sigma''^2 \lambda' \, \Omega''(f) = f + K_1 \exp(\omega_1 f) + K_2 \exp(\omega_2 f) \tag{4.4'}$$

where the roots ω_1 and ω_2 follow from the characteristic equation

$$\tfrac{1}{2} (\lambda' \sigma''^2) \, \omega^2 + (\lambda' f') \, \omega - 1 = 0$$

and the constants K_1 and K_2 follow from the smooth-pasting conditions $\Omega'(f_L)=\Omega'(f^U)=0$ where $e^U=\Omega(f^U)$ and $e_L=\Omega(f_L)$. When the constant component of the fundamental is zero ($f'=0$) *and* the band is symmetric ($e^U=-e_L$), one obtains the symmetric solution[11]

$$e = \Omega(f) = f - [\omega \cosh(\omega f^U]^{-1} \sinh(\omega f), \quad \omega \equiv (2/\lambda')^{½}/\sigma''. \tag{4.5}$$

Clearly, the solution $e=\Omega(f)$ has the S-shape familiar from the model of Krugman (1991). As the accommodation coefficient (β) approaches the value corresponding to a peg ($1-\delta$), ω increases and the S-shape becomes less pronounced in the sense that the implicit band on prices becomes wider. This classical model implies a U-shaped unconditional density function of the exchange rate (cf., Svensson, 1991a). To account for the hump-shaped unconditional density functions exhibited by EMS exchange rates, one has to allow for a stronger form of mean reversion.[12] This is what Section 5 sets out to do.

Krugman (1991) assumes purchasing power parity and thus a constant real exchange rate. Here the real exchange rate is stochastic,

$$p^*+e-p=[\eta(1-\beta)+\delta\lambda]^{-1}n(1-\beta)[\lambda' i^*+K_1\exp(\omega_1 f)+K_2\exp(\omega_2 f)]+\delta^{-1}(y^F+\epsilon), \tag{4.6}$$

even when there is no band on the nominal exchange rate ($K_1=K_2=0$).

5 Exchange rate bands, monetary accommodation and unemployment

5.1 Analysis

To allow for a stronger form of mean reversion in the fundamental, labour market inflexibility is introduced. It is thus necessary to solve the system (2.9)-(2.10) with the nonlinear intervention policy arising from exchange rate bands and intramarginal monetary accommodation. To do this, one postulates instead of the linear saddlepath solution (2.11) a twice differentiable function for the solution:

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[11] Clearly, when there is a laxer monetary policy at home than abroad, the constant component of the fundamental is positive ($f'>0$), the currency has a tendency to be weak, and thus more of the probability mass is concentrated near the upper part of the band.

[12] Froot and Obstfeld (1991b) discuss the effects of an Ornstein-Uhlenbeck process for the fundamental in the Krugman (1991) model.

$$e + p^* - \mu = \Omega\ [p - \mu]. \tag{2.11'}$$

Use of Ito's Lemma, $d\Omega = \Omega'\ dp + \frac{1}{2}\ \sigma^2\ \Omega''\ dt$, yields a second-order nonlinear differential equation (cf., Miller and Weller, 1989, 1991):

$$\tfrac{1}{2}\ \sigma^2\ \Omega''(p) + \phi(\eta+\lambda)^{-1}\ \{-[\delta\lambda+\eta(1-\beta)]\ p + \delta\lambda\ \Omega(p)\}\ \Omega'(p)\ - \\ (\eta+\lambda)^{-1}\ [(1-\delta-\beta)\ p + \delta\ \Omega(p)] = 0. \tag{5.1}$$

This equation yields a time-invariant relation between exchange rates and prices. Note that there are exactly two linear solutions which correspond exactly to the stable and the unstable arm of the saddlepath solution of the dirty float model discussed in sections 2 and 3. These linear solutions are, of course, not compatible with the presence of finitely wide exchange rate bands.

Corresponding to the exchange rate bands, one can use the solution to (5.1) to define bands on producer prices ("fundamentals"), namely $p^U=\Omega^{-1}(e^U)$ and $p_L=\Omega^{-1}(e_L)$. For low degrees of monetary accommodation, $\beta<1-\delta$, $\Omega'<0$ and thus $p^U<p_L$. For high degrees of monetary accommodation, $\beta>1-\delta$, $\Omega'>0$ and thus $p^U>p_L$. The solution for these two cases yields an inverted S-shape and a normal S-shape, respectively, and are both presented in Figure 11. The slopes of the nonlinear saddlepath at the origin are exactly the same as the slope of the linear saddlepath under a dirty float (see Section 3). To pin down a unique solution, one needs to impose the smooth-pasting conditions (cf., Dixit, 1989; Krugman, 1991; Miller and Weller, 1989, 1991) in order to avoid one-way bets by market participants. If these conditions do not hold at p_L and p^U, the expected change in the exchange rate would discretely jump at these boundaries which is incompatible with the absence of unexploited arbitrage opportunities. Clearly, the nonlinear saddlepath has maximum absolute value of slope at the origin and zero slope at the boundaries.

Because the market anticipates a regime switch at the boundaries of the band, the solution bends and becomes horizontal as the economy approaches the boundaries. This is what is known as the 'honeymoon effect' and becomes stronger as the band on exchange rates narrows. Clearly, a higher variance σ^2 leads to a stronger honeymoon effect as a higher coefficient of $\Omega''(p)$ in (5.1) introduces more nonlinearity. A higher value of λ adds to the forward-looking character of the solution and, typically, the exchange rate solution bends more towards the central parity, i.e. there is a stronger honeymoon effect.

Figure 12 shows how the width of the band on the price level depends on the accommodation coefficient. For low degrees of accommodation, the upper bound e^U translates into a lower bound on the price level, $p^U<0$, which increases towards infinity as the accommodation coefficient approaches the value corresponding to a peg. The point is that very low prices induce an expansion of the real money supply and thus a depreciation of the currency, possibly forcing it beyond the upper bound. For high degrees of accommodation, the upper bound e^U translates into an upper bound on the price level, $p^U>0$, which decreases from infinity as the accommodation coefficient approaches unity. The inward shift in the LM-curve arising from high prices is now attenuated by accommodation, but the IS-curve shifts back a lot. The result is an incipient interest rate differential in favour of the foreign country and pressure on the currency to depreciate on impact, possibly forcing it outside the band. An increase in λ increases the band on prices. An increase in ϕ increases (decreases) the band on prices when β is less than (exceeds) $1-\delta$. An increase in σ^2 leads for a given exchange rate band to a wider band on prices.

The accommodation coefficient of the monetary authorities only changes at the boundaries of the exchange rate band. In other words, the monetary authorities employ a nonlinear accommodation rule:

$$m = \beta\ p, \text{ if } e_L<e<e^U \text{ (i.e. } p^U<p<p_L \text{ for } \beta<1-\delta,\ p_L<p<p^U \text{ for } \beta>1-\delta),$$

$$m = (1-\delta)\ p + \delta\ e_L, \text{ if } e\leq e_L \text{ (i.e. } p\geq p_L \text{ for } \beta<1-\delta,\ p\leq p_L \text{ for } \beta>1-\delta),$$

$$m = (1-\delta)\ p + \delta\ e^U, \text{ if } e\geq e^U \text{ (i.e. } p\leq p^U \text{ for } \beta<1-\delta,\ p\geq p^U \text{ for } \beta>1-\delta).$$

Figure 13 sketches the money supply rule, both for high and low degrees of monetary accommodation. At the upper (lower) bound for the exchange rate, there is a danger that the currency depreciates (appreciates) outside the band. The monetary authorities thus implement a discrete contraction (expansion) of the money supply which raises (depresses) the interest rate and prevents the exchange rate from moving outside the band.

5.2 Simulated unconditional variances

To compare the asymptotic unconditional variances of the model under a nonlinear intervention rule with those discussed in section 3, it is necessary to resort to numerical Monte Carlo simulations (see Appendix). The reason is that one cannot obtain an analytical closed-form solution for the exchange rate when it is inside the band, and it is difficult to find expressions for the time spent at either the upper or the lower boundary of the band. The parameter values that have been used in all the simulations are $\eta=\delta=\phi=\lambda=\frac{1}{2}$ and a symmetric band of 2.25%. Note that the existence of the distribution functions requires, as shocks are normally distributed, that the process for the exchange rate is stationary and ergodic.

Figures 14 and 15 portray the simulated distributions of the nominal exchange rate and interest rate differential under various degrees of accommodation. When the degree of accommodation is not very strong or close to the PPP rule, the simulated density functions of the exchange rate are U-shaped and the exchange rate spends a relatively large proportion of the time near the boundaries of the band. However, when the accommodation coefficient approaches the value corresponding to a peg, these density functions are hump-shaped and the exchange rate spends a relatively large proportion of the time near the central parity.[13] The latter case clearly resembles the one found for EMS exchange rates (see Figure 1). This strongly suggests that countries such as France and the Netherlands stabilise their exchange rate through intramarginal as well as inframarginal interventions.[14] As far as the interest rate is concerned, most of the probability mass is concentrated around the foreign interest rate. Furthermore, Figure 7 shows that the variance diminishes as the accommodation coefficient approaches the value corresponding to a peg. Note that, as one approaches the PPP rule, the variance of the interest rate approaches zero under an unrestricted float but approaches a positive value under a band. In fact, the variance of the interest rate under a PPP rule first increases and then decreases as the bandwidth, and finally is zero for an infinite

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[13] The reason is the combined effect of mean reversion arising from the Phillips curve, the band on prices getting wider and consequently the relative amount of time spent between the margins getting larger as β approaches $1-\delta$

[14] This explanation offers an alternative to the one based on repeated realignments offered by Bertola and Caballero (1990).

bandwidth.

Mussa (1986) noticed that for many countries the volatility of real exchange rates declines as the volatility of nominal exchange rates declines. Obviously, this phenomenon can only be explained within the context of a Keynesian model with nominal wage sluggishness. Figures 8 and 9 illustrate that this stylised fact can be explained when $\beta<1-\delta$, i.e. for low accommodation coefficients a reduction in the bandwidth is accompanied by a reduction in the variance of the real as well as of the nominal exchange rate. However, for high accommodation coefficients a reduction in the bandwidth raises the variance of the real exchange rate.

6 Welfare losses arising from a nominal exchange rate band

Within the class of linear dynamic models and quadratic preferences, it is never optimal to have a nonlinear policy rule (e.g. Davis, 1977, Chapter 5). Since the imposition of a exchange rate band corresponds to a nonlinear accommodation rule (see Figure 13), it is clear that exchange rate bands induce welfare losses.[15] A key question is how much worse one is off with a band on the nominal exchange rate instead of an unrestricted dirty float. Figure 10 confirms that the policy frontier corresponding to an unrestricted dirty float lies entirely below the ones corresponding to an exchange rate band and gives an idea of these losses. As the bandwidth approaches infinity, the policy frontier approaches the one corresponding to an unrestricted dirty float. If the bandwidth goes to zero, the policy frontier reduces to a point, i.e. the point corresponding to the peg. In fact, all the policy frontiers go through this same point irrespective of the bandwidth.

Clearly, the model of sections 2 and 3 is a special case of the model of section 5, namely the case where the exchange rate band is infinitely wide. In general, the monetary authorities can choose two instruments, i.e. the accommodation coefficient (β) and the bandwidth ($e^U - -e_L$). Given the availability of the first instrument, it is never optimal to use the second. To put it differently, the optimal bandwidth from a stabilisation point of view is infinite. However, one can phrase a second-best question:

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[15] Of course, it may be possible to make a welfare case for exchange rate bands on credibility rather than on stabilisation grounds but that is beyond the scope of the present paper. Here it is assumed that the monetary authorities commit themselves to a simple accommodation rule, which can readily be understood by private agents.

What are the reductions in welfare losses one can achieve with having a finite exchange rate band given that one has a given degree of monetary accommodation of the form (2.6) within band? Figures 4, 6 and 10 provide an answer.

Figure 4 plots the variance of output against the accommodation coefficient for various bandwidths. When the accommodation coefficient corresponds to the value of a peg, the bandwidth is irrelevant. In fact, the various plots pivot around $\beta=1-\delta$. When one decreases the width of the band from infinity (an unrestricted dirty float) to zero (a peg), the plot of var(y) bends and becomes more and more horizontal. For low levels of accommodation, var(y) under a band is less than that under an unrestricted dirty float. For high levels of accommodation, var(y) under a band is greater than that under a dirty float. The intuition of the shape of these plots is that, for a given β, the variance under a band is roughly a weighted average of the variance under an unrestricted dirty float and under a peg.

Figure 6 plots var(q) versus β for various bandwidths. Again, the plots pivot around $\beta=1-\delta$. When the bandwidth becomes smaller, var(q) increases for low and decreases for high levels of accommodation. The plots for var(p) are given in Figure 5. They are qualitatively the same, except that var(p)>var(q) for all values of β.

If the authorities can only use a little accommodation in between the bands, one expects that the higher the weight the authorities attach to CPI rather than output stability (γ), the lower the chance that the authorities wish to impose a narrow exchange rate band. Figures 4, 6 and 10 confirm this, because under an unrestricted dirty float or a wide band the variance of output is higher and the variance of the CPI is lower than under a narrow band.

These figures also show that, with high degrees of accommodation, an unrestricted dirty float or a wide band leads to less volatility of output and more volatility of the CPI. In that case, right-wing authorities are more likely to impose a narrow exchange rate band.

Figure 10 shows the policy frontier under a ±2.25% and a ±6% band. In a sense, increasing accommodation when $\beta<1-\delta$ is like reducing bandwidth, and when $\beta>1-\delta$ it is like increasing bandwidth. In particular, for a given value of β below (above) $1-\delta$, a reduction in bandwidth depresses (raises) var(y) and raises (depresses) var(q).

Perhaps, a more interesting second-best question is how the optimal degree of accommodation changes as the band narrows. For low degrees of accommodation ($\beta<1-\delta$), the variance of the CPI rises and that of output falls as the band narrows. It may then be optimal to reduce the degree of intramarginal accommodation, because this reduces the variance of the CPI and increases that of output. For high degrees of accommodation, the variance of the CPI falls and that of output rises as the band narrows so that if the authorities compensate they choose to have a higher degree of intramarginal accommodation.

7 Concluding remarks

This paper studied a small open economy under a dirty float. It analysed the optimal degree of monetary accommodation and showed that this increases when the relative weight on employment rather than on CPI stability increases. More flexible labour markets induce right-wing governments to move towards a cleaner float and left-wing governments towards a PPP exchange rate rule. A clean float, a peg or a PPP exchange rate rule are seldom optimal. This paper also extended the Miller-Weller-Krugman model to allow for monetary accommodation of price shocks within the band. An interesting feature is that when the accommodation coefficient approaches the value corresponding to a peg, the unconditional density function of the exchange rate is hump-shaped rather than U-shaped. This finding explains a stylised fact of EMS exchange rates which cannot be explained with the classical Krugman (1991) model or the more Keynesian Miller-Weller (1991) model, because these studies do not allow for intra-marginal interventions. For low values of the accommodation coefficient, the model can also explain a stylised fact commented on by Mussa (1986), that is as the exchange rate band narrows the variance of both the real and the nominal exchange rate diminish.

It is hard to make a case for exchange rate bands, since the welfare loss can more effectively be reduced by means of a linear accommodation rule without bands. This suggests that the welfare case for exchange rate bands cannot be made on stabilisation grounds alone. Future research may therefore be directed at investigating whether a welfare case for exchange rate bands can be made on credibility rather than on stabilisation grounds. Such an analysis may trade off the benefits of a simple fixed rule in terms of enhanced credibility against the costs in terms of reduced scope for stabilisation (cf., Giavazzi and Pagano, 1988).

References

Alogoskoufis, G.S. (1991), Monetary accommodation, exchange rate regimes and inflation persistence, DP No. 503, CEPR, London.

Bergstrom, A.R. (1984), Continuous time stochastic models and issues of aggregation over time, in Z. Griliches and M.D. Intrilligator (eds.), *Handbook of Econometrics, Vol. II*, North-Holland, Amsterdam.

Bertola, G. and R.J. Caballero (1992), Target zones and realignments, *American Economic Review*, forthcoming.

Bertola, G. and L.E.O. Svensson (1991), Stochastic devaluation risk and the empirical fit of target zone models, DP No. 513, CEPR, London.

Davis, M.H.A. (1977), *Linear Estimation and Stochastic Control*, Chapman and Hall, London.

Davis, P.J. and P. Rabinowitz (1975), *Methods of Numerical Integration*, Academic Press, New York.

Dixit, A. (1991), A simplified treatment of the theory of optimal regulation of Brownian motion, *Journal of Economic Dynamics and Control*, 15, 657-673.

Dornbusch, R. (1976), Exchange rate expectations and monetary policy, *Journal of Political Economy*, 84, 1161-1176.

Dornbusch, R. (1982), PPP exchange-rate rules and macroeconomic stability, *Journal of Political Economy*, 90, 158-165.

Duffie, D. and K.J. Singleton (1988), Simulated moments estimation of diffusion models of asset prices, mimeo, Stanford University.

Froot, K. and M. Obstfeld (1991a), Exchange-rate dynamics under stochastic regime shifts: A unified approach, *Journal of International Economics*, forthcoming, DP No, 522, CEPR, London.

Froot, K. and M. Obstfeld (1991b), Stochastic process switching. Some simple solutions, *Econometrica*, 59, 241-250.

Giavazzi, F. and M. Pagano (1988), The advantage of tying one's hands: EMS discipline and central bank credibility, *European Economic Review*, 32, 5, 1055-1082.

Harrison, J.M. (1985), *Brownian Motion and Stochastic Flow Systems*, John Wiley, New York.

Karlin, S. and H.M. Taylor (1981), *A Second Course in Stochastic Processes*, Academic Press, New York.

Klein, M.W. (1990), Playing with the band: Dynamic effects of target zones in an open economy, *International Economic Review*, 31, 4, 757-772.

Krugman, P.R. (1991), Target zones and exchange rate dynamics, *Quarterly Journal of Economics*, CVI, 669-682.

Lewis, K.K. (1990), Can managed float interventions make exchange rate behaviour resemble a target zone?, mimeo, New York University.

Miller, M. and P. Weller (1989), Solving stochastic saddlepoint systems: A qualitative treatment with economic applications, DP No. 308, CEPR, London.

Miller, M. and P. Weller (1991), Exchange rate bands with price inertia, *Economic Journal*, 101, 1380-1399.

Mussa, Michael (1986), Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications, in K. Brunner and A. Meltzer (eds.), Carnegie-Rochester Conference Series on Public Policy, 25, 117-124.

Svensson, L.E.O. (1991a), Target zones and interest variability, *Journal of International Economics*, 31, 27-54.

Svensson, L.E.O. (1991b), The term structure of interest rate differentials in a target zone: Theory and Swedish data, *Journal of Monetary Economics*, 28, 1, 87-116.

Appendix: Monte Carlo simulations

The simulation technique is based on Duffie and Singleton (1988). Prior to simulation, the second-order ordinary differential equation (5.1) is solved numerically for the function $\Omega(p)$. Given roughly 200 points of this function between the boundaries of the band, intermediate points were found using linear interpolation of the two nearest points. Third-order polynomial interpolation (Davis and Rabinowitz, 1975) yields results that are hardly different. To simulate the structural continuous-time model, it must be discretised. The parameter values that have been chosen correspond to a unit time interval. First, given p_t, e_t and a draw from the normal distribution for the additive shock, p_{t+1} can be calculated as a linear combination of these three variables. Second, one can calculate $e_{t+1}=\Omega(p_{t+1})$ from the numerical solution $\Omega(p)$. It is straightforward to calculate the simulated values of other endogeneous variables as well.

Clearly, this discretisation is much too coarse and estimates of asymptotic variances based on this discretisation are inconsistent. Better approximations are obtained if the model is simulated at a higher frequency and a finer grid is used. Hence, in practice each unit of time is divided into n parts and n in-between simulations are performed over each of the T unit time intervals. Obviously, the parameter values are adjusted to allow for the higher sampling frequency. Similarly, the variance is adjusted to allow for the higher frequency, i.e. σ/n replaces σ. Although more sophisticated schemes are available, a simple first-order Euler scheme has been used in the simulations presented in this paper. Duffie and Singleton (1988) show that consistent estimation of the variances of the variables requires a minimal ratio n/T, at which both n and T should go to infinity. In fact, for the special processes considered here, the second-order Milshtein scheme reduces to the Euler scheme.

The set-up of the Monte Carlo simulation procedure is as follows. From a long time series, N series of T observations are drawn, making sure to throw away a number of observations between subsequent sequences of T observations. A grid of n is used for the in-between simulations. In practice, typical values were $n=25$, $N=10$ and $T=1000$.

If n tends to infinity, the discrete approximation to the continuous-time distribution becomes exact. Because all variables are defined to have an asymptotic mean equal to zero and n is chosen to be relatively large, the test statistic

$$s_{x,i}^2 \equiv (x_{1i}^2 + x_{2i}^2 + \ldots + x_{Ti}^2)/T, \quad i=1,2,\ldots,N$$

is approximately an unbiased estimator of the unconditional variance of the variable x. Clearly,

$$\hat{s}_x^2 \equiv (s_{x,1}^2 + s_{x,2}^2 + \ldots s_{x,N}^2)/N$$

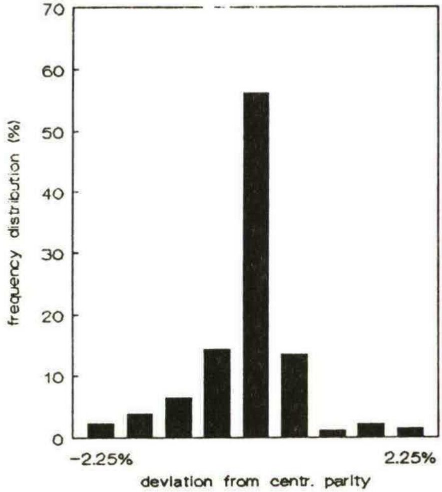
is also an estimator of the unconditional variance of x. The statistic

$$[(s_{x,1}^2-\hat{s}_x^2)^2 + \ldots + (s_{x,N}^2-\hat{s}_x^2)^2]/[N(N-1)]$$

is approximately an unbiased estimator of the variance of $\hat{s}_x^2$, given that the number of observations thrown away between subsequent sequences of T observations is large enough to rule out serial correlation between the $s_{x,i}^2$.

Figure 1: Histograms of selected EMS exchange rates

(a) Guilder-Deutschemark rate



(b) Franc-Deutschemark rate

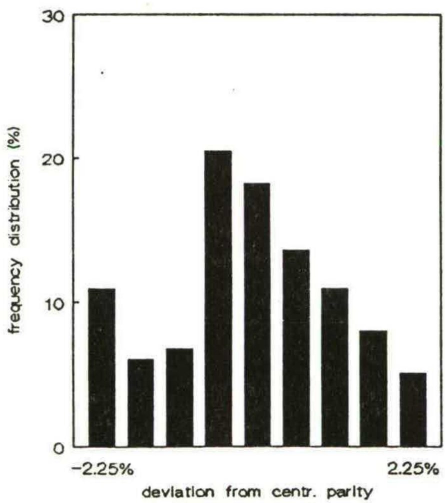
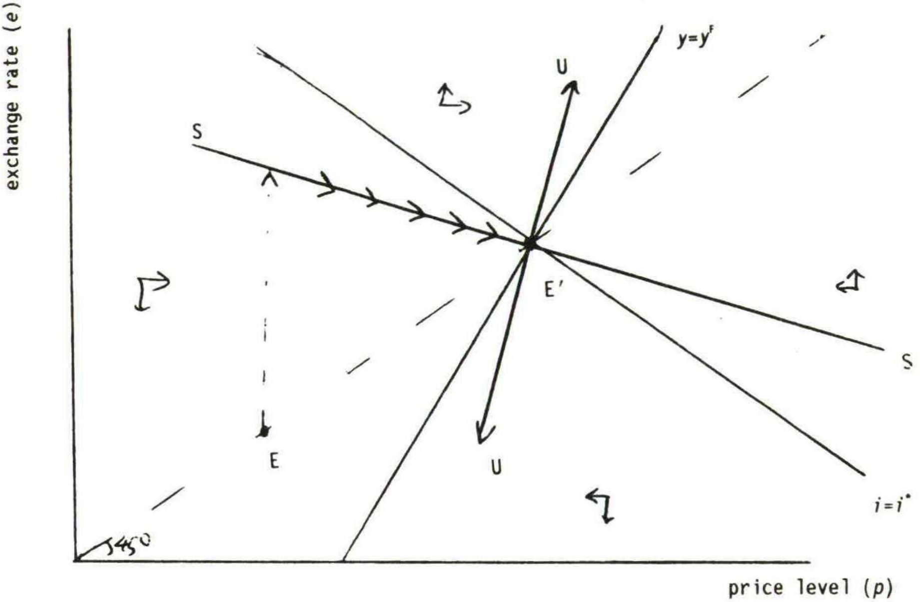


Figure 2: A permanent increase in the money supply under a dirty float

(a) Low degree of monetary accommodation ($\beta<1-\delta$)



(b) High degree of monetary accommodation ($\beta>1-\delta$)

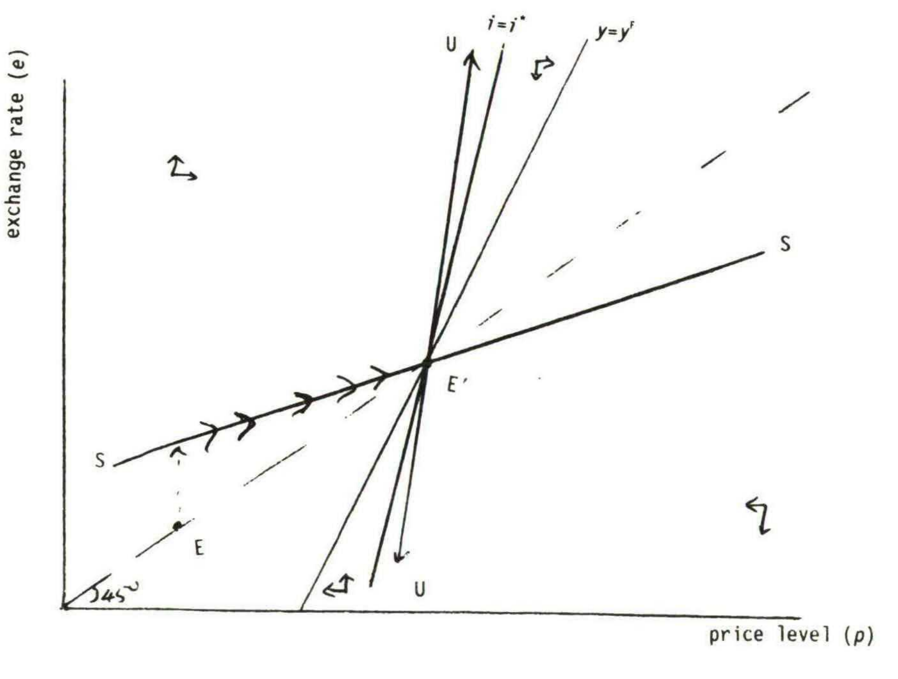


Figure 3: Sensitivity of the slope of the saddlepath under a dirty float

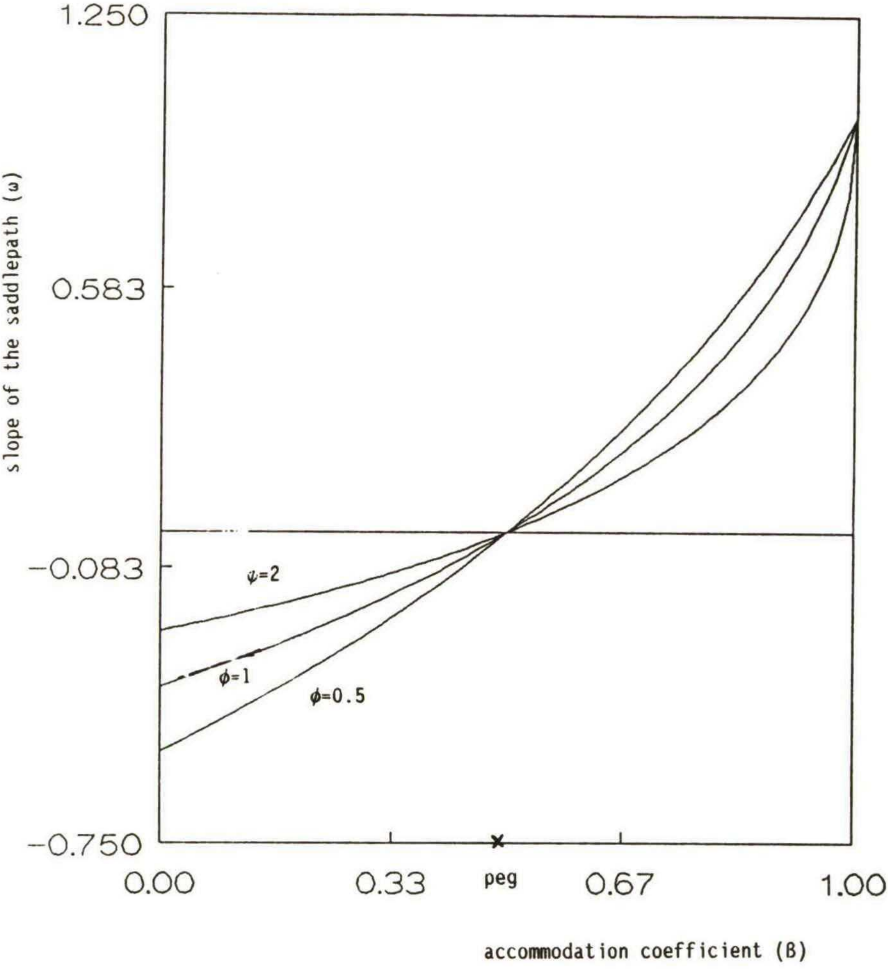


Figure 4: Variance of output under various bandwidths

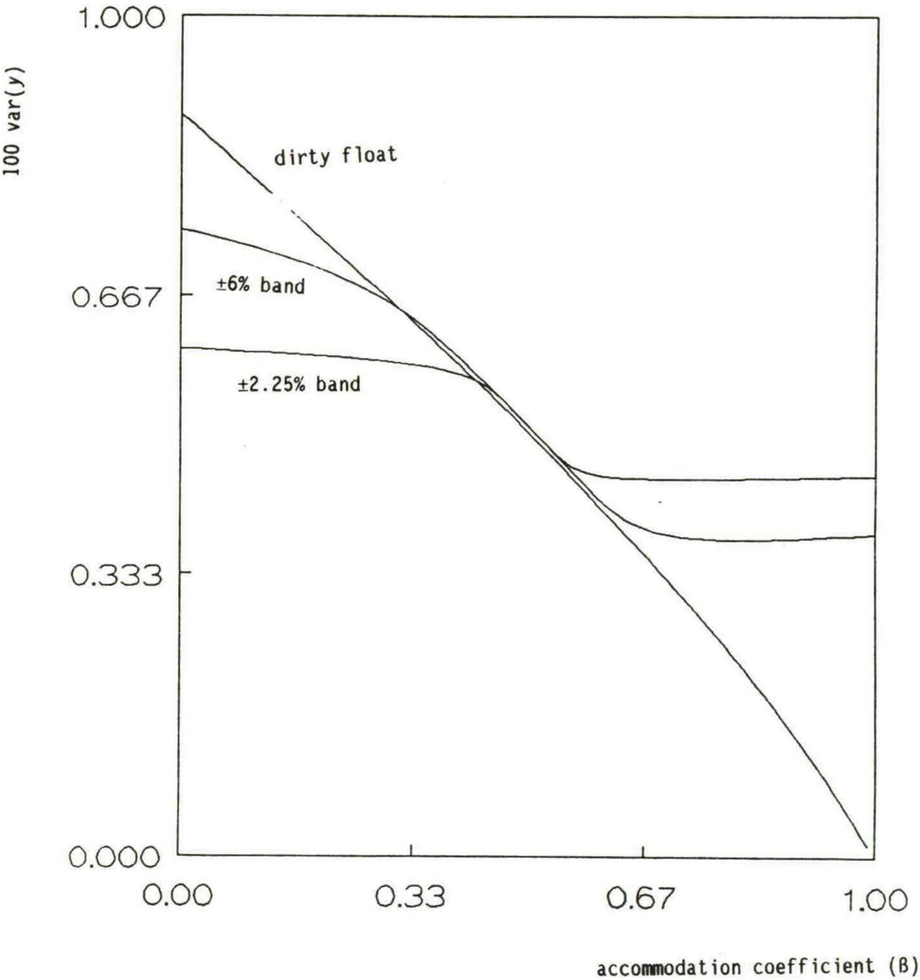


Figure 5: Variance of the price level under various bandwidths

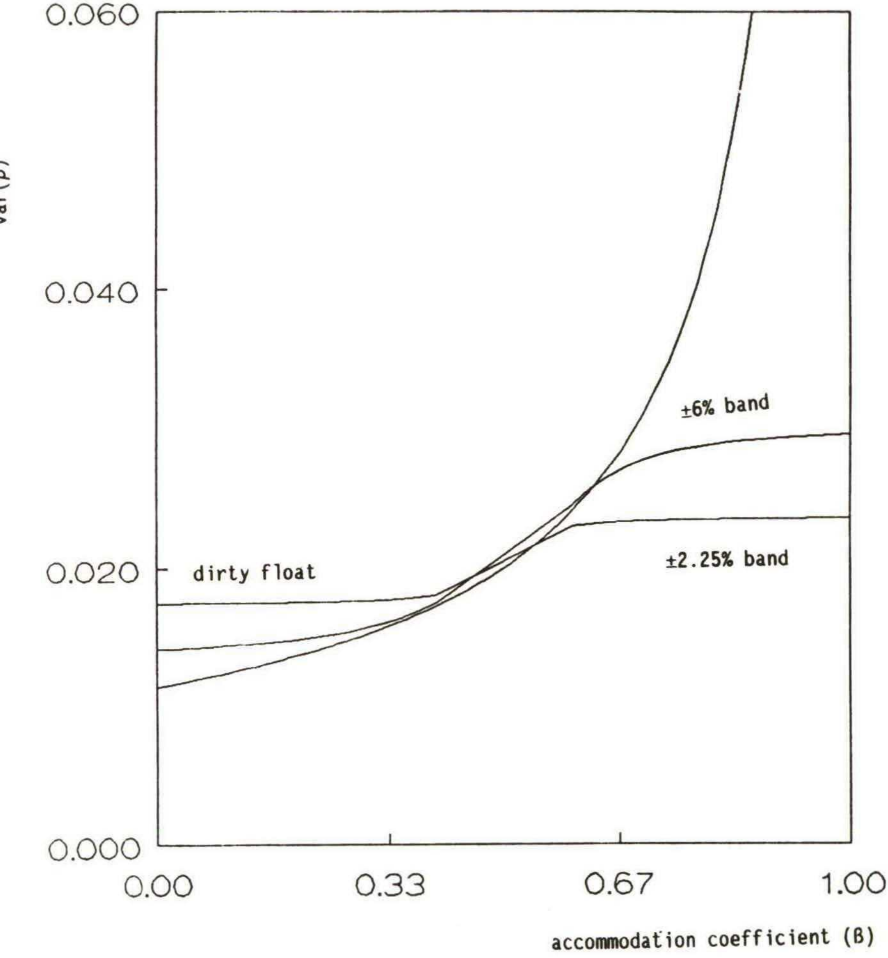


Figure 6: Variance of the CPI under various bandwidths

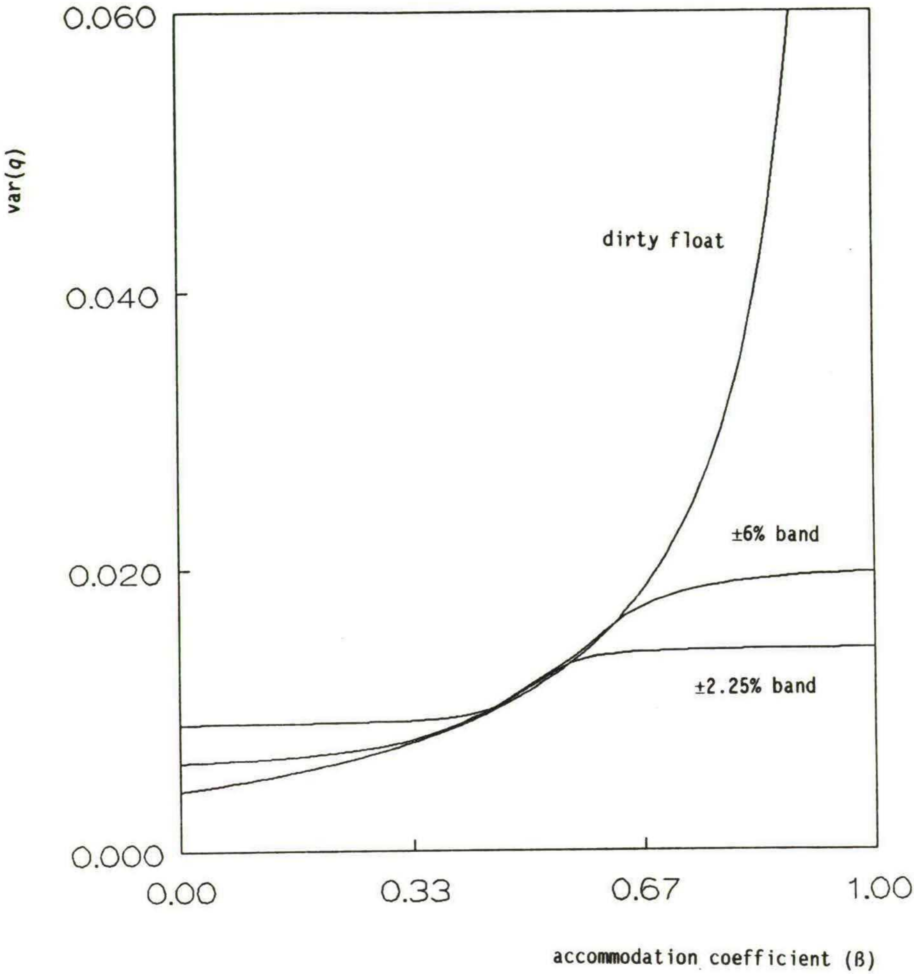


Figure 7: Variance of the nominal interest rate under various bandwidths

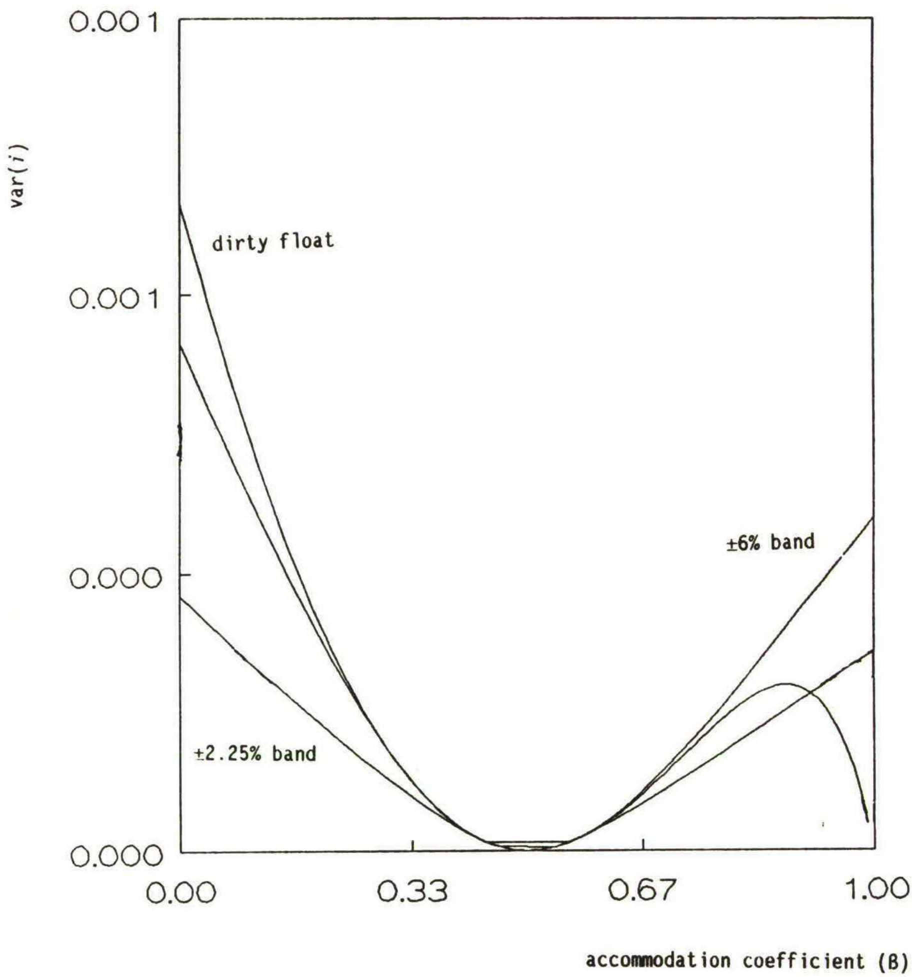


Figure 8: Variance of the nominal exchange rate under various bandwidths

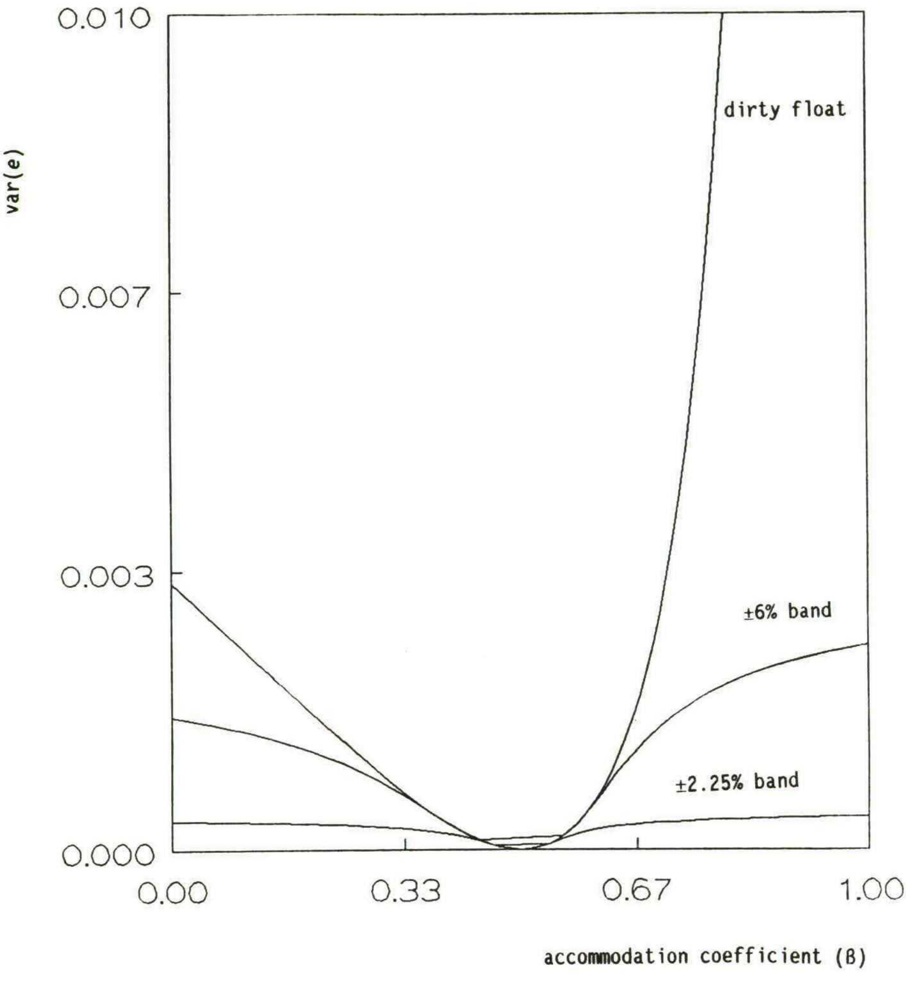


Figure 9: Variance of the real exchange rate under various bandwidths

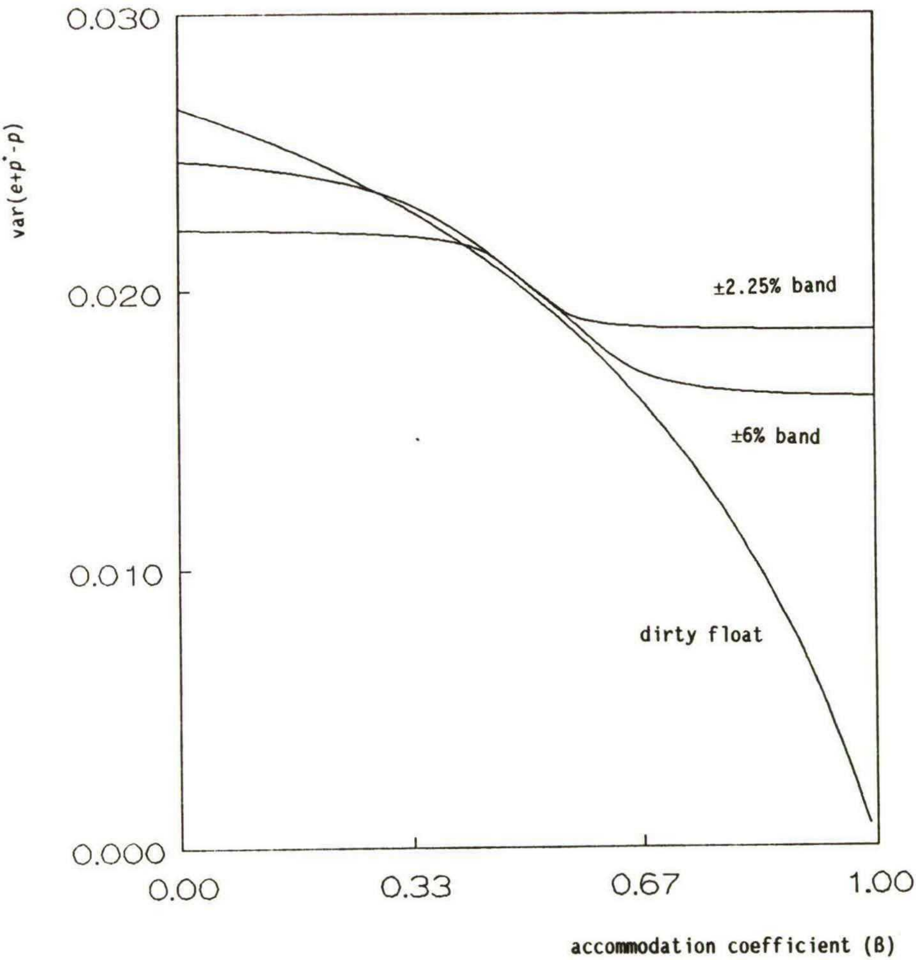


Figure 10: Optimal monetary accommodation under a dirty float

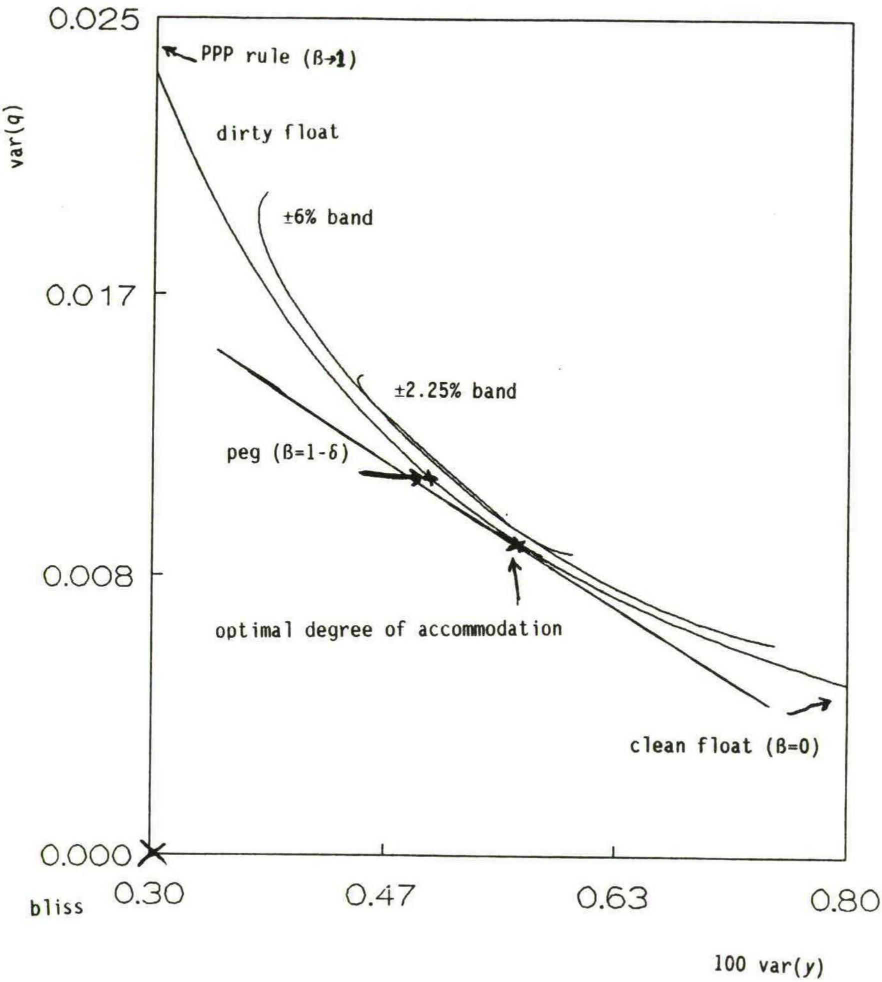


Figure 11: Exchange rate bands and monetary accommodation

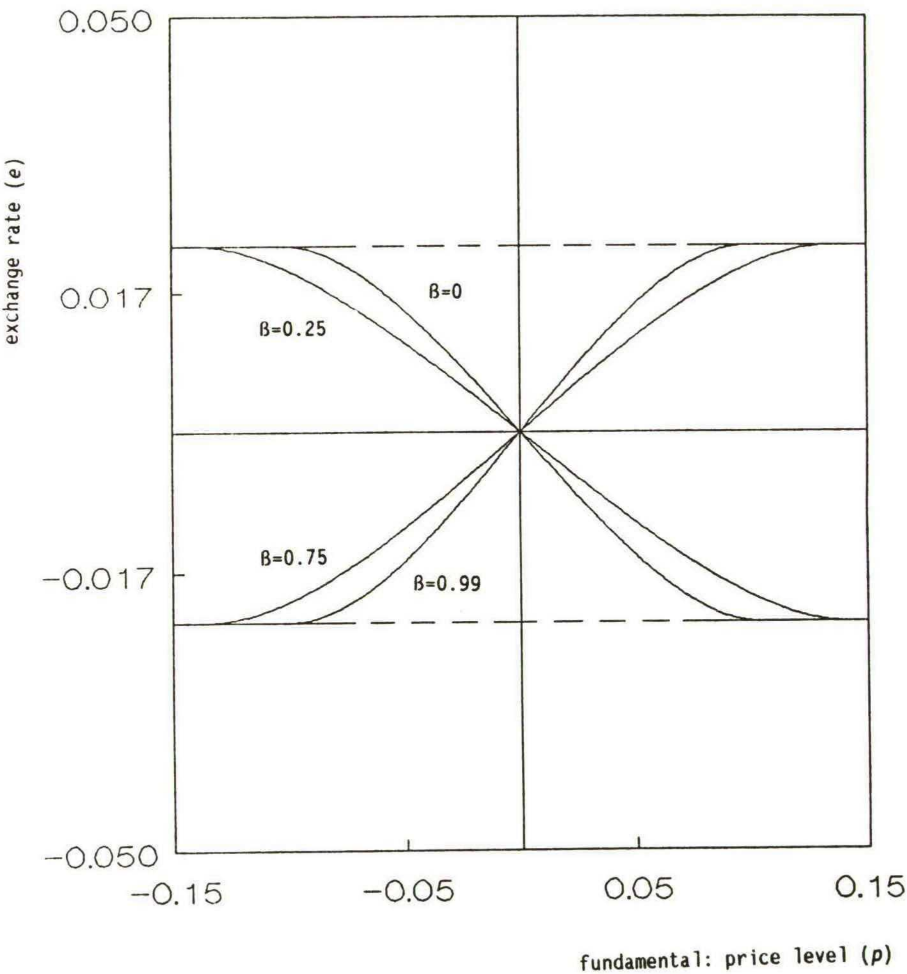


Figure 12: Price bands and exchange rate bands

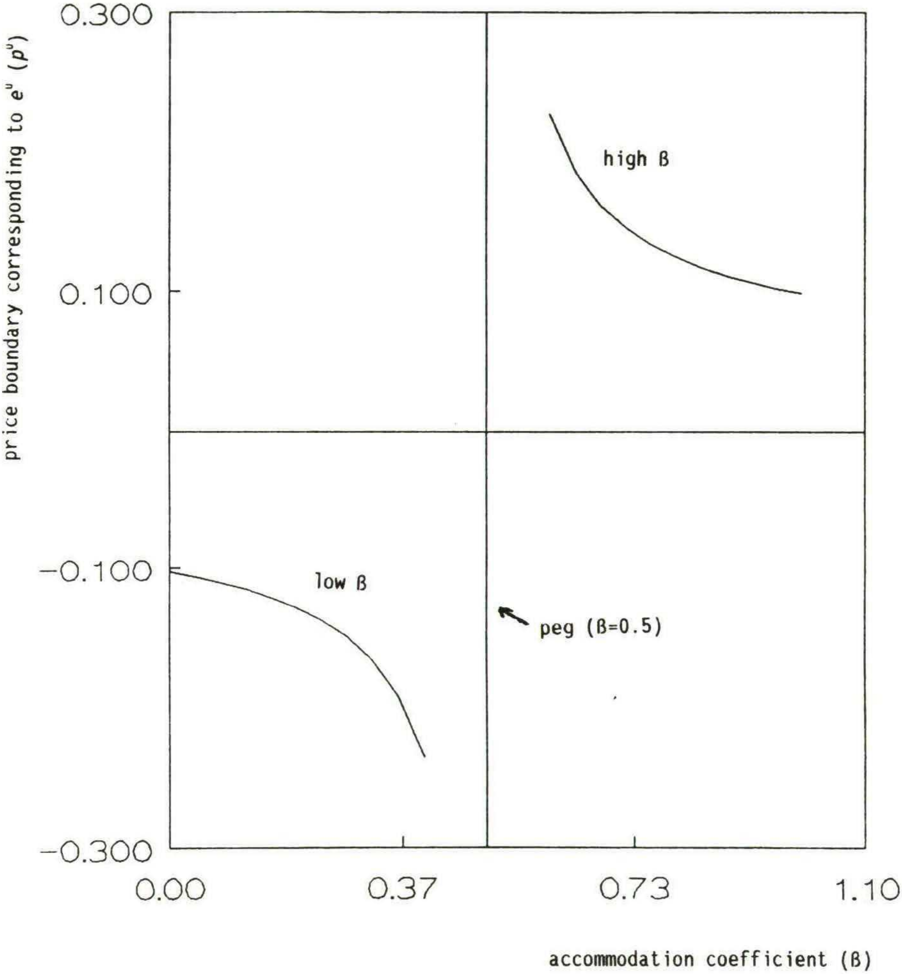
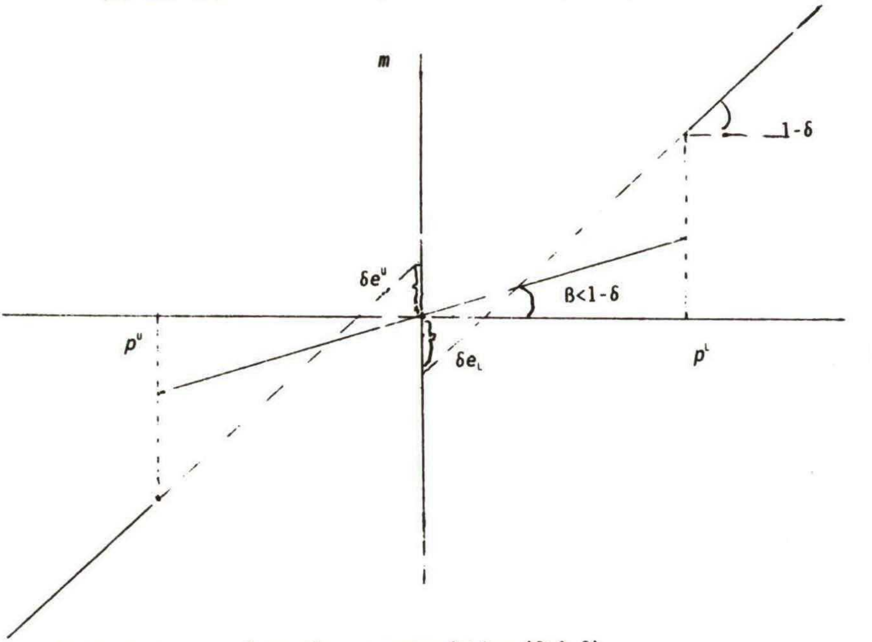


Figure 13: Nonlinear money supply rules

(a) Low degree of monetary accommodation ($\beta<1-\delta$)



(b) High degree of monetary accommodation ($\beta>1-\delta$)

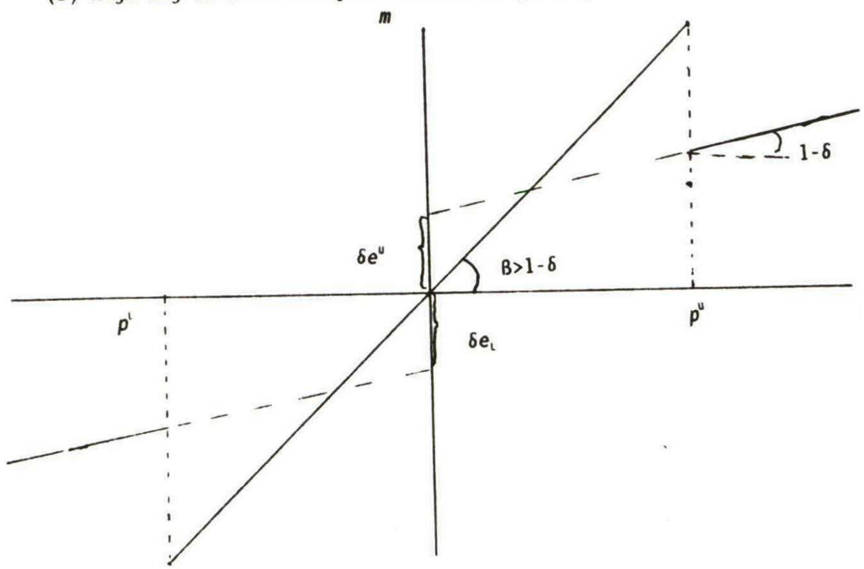
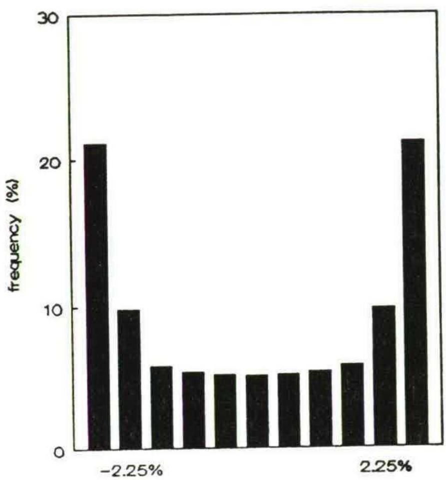
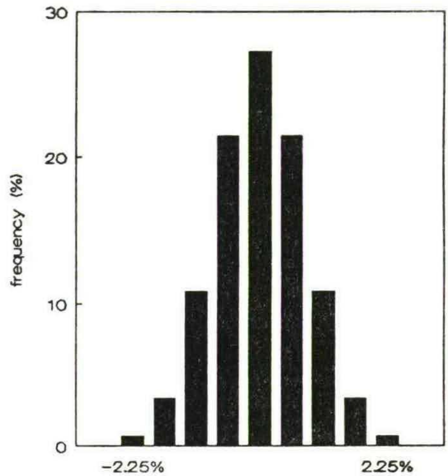


Figure 14: Simulated distribution of the exchange rate under a band and various degrees of accommodation

(a) A clean float (ß=0)



(b) Almost a peg (ß=0.46)



(c) Almost a PPP exchange rate rule (ß=0.98)

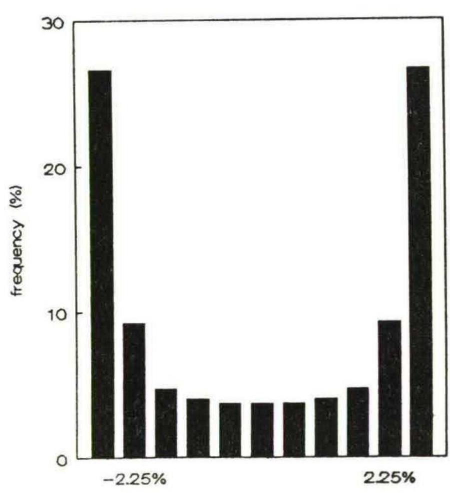
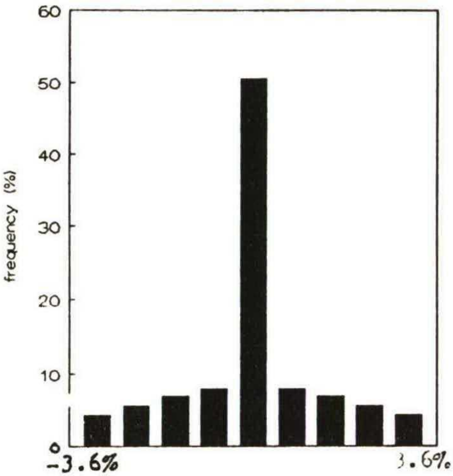
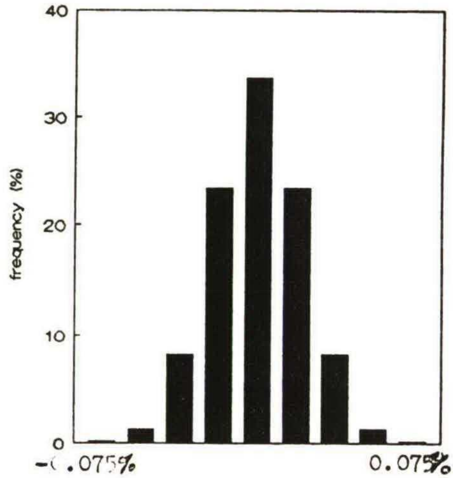


Figure 15: Simulated distribution of the interest rate differential under a band and varying degrees of accommodation

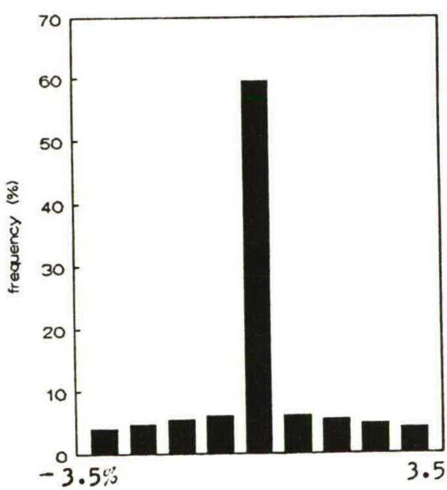
(a) A clean float (ß=0)



(b) Almost a peg (ß=0.46)



(c) Almost a PPP exchange rate rule (ß=0.98)



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